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ABSTRACT

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ABSTRACT

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GERALD KULM, Purdue University

The study tested the hypothesis that concrete operational subjects learn better from a laboratory-based concrete activity approach than would formal subjects. Four units of mathematics over a seven week period were studied by 89 subjects divided into lecture and laboratory groups. A 2(treatment) by 2(cognitive level) design was used to analyze achievement on four unit tests, a comprehensive test, and a retention test. Significant interactions on two unit tests, and the achievement and retention tests were found in ANOVAS indicating that formal subjects learned better in the laboratory and concrete subjects learned better in the lecture approach.

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Laboratory and Lecture Approaches to Mathematics Instruction for Concrete and Formal College Students

GERALD KULM, Purdue University

A substantial amount of evidence seems to indicate that many college freshmen have not yet reached the Piagetian level of formal thinking. For example, studies by Elkind (1962), McKinnon and Renner (1971), and Tower and Wheatley (1971) indicated that 40 to 75 percent of college freshmen were functioning at Piaget's concrete operational level in one or more areas. Lawson (1974) has suggested that there are certain abstract concepts that are understandable to students at the concrete operational level, while other concepts can only be learned at the formal level. Further, he showed a direct relationship between the learning of formal concepts and the level of cognitive development.

Many of the concepts in mathematics are formal in nature. One approach to making these formal concepts attainable by concrete subjects is to present them through enactive or iconic representations, enabling the subject to progress to a more symbolic and formal understanding. The sequence of mathematics instruction which moves from the use concrete or semi-concrete activities and materials to symbolic rules, definitions or formulas is known as the mathematics laboratory approach. The approach is largely student-centered, involving small group and individual exploration. This activity, according to Dienes (1963) provides the student with a mathematical imagery, eventually enabling the manipulation of symbols without the aid of concrete materials.

Paper presented at American Educational Research Association Annual Meeting, New York, April 6, 1977.

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Previous research studies in the use of the laboratory approach at the college level have not used cognitive level as an independent variable. These studies have found no significant achievement differences between laboratory and lecture groups (Cathcart 1975, Fuson 1975, Warkentin 1975). A further difficulty with past studies has been that the content for the treatment and control groups has not been identical, making it impossible to compare achievement results (Fitzgerald 1968, Warkentin 1975).

The purpose of the present study was to determine the effectiveness of a laboratory approach for formal and concrete operational subjects who were given identical instructional objectives and achievement measures. It was hypothesized that a laboratory approach would be more effective for concrete subjects and that a lecture approach would be more effective for formal subjects.

Methods

Subjects: The subjects were 89 freshmen in a first mathematics course for elementary teachers.

Instructional Materials: Two parallel sets of instructional booklets were prepared for each of the five units studied during the experiment. The booklets contained the rationale, behavioral objectives, learning activities, and a self test. The learning activities consisted of textbook readings and exercises, differing only in the classroom activities. Classroom lectures for one group were replaced by laboratory activities in the experimental group. The laboratory work was done through group work on worksheets prepared by the experimenter. Manipulative materials were used in conjunction with the worksheets to promote discovery of concepts and relationships. The same two graduate instructors taught both groups. The instructors were available to answer questions

and provide suggestions for proceeding through the worksheets. At the end of each unit, usually four or five class sessions, a ten item quiz was given to both groups over the unit objectives. A brief outline of each of the units is given in Table 1.

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Insert Table 1 about here
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At the end of the eight week experiment, a comprehensive achievement test over the five units was given. Eight weeks later, the achievement test was given again as a part of a final examination to measure retention.

Formal Reasoning Test: During the first week of classes, an eight-item open-response test was administered to assess cognitive level. Two items in each of four areas were written to determine subjects' ability in proportional thinking, syllogistic reasoning, combinatorial thinking and field dependence. The subjects' responses to the items were scored either correct or incorrect, resulting in a maximum possible score of 8. Examples of the items are given below.

Proportional thinking item:

Teacher Puzzle

A teacher gives a math problem to three different classes. Shown below is a summary of the results.

Class	Number of Students working problem correctly	Number of students working problem incorrectly
1	20	10
2	22	20
3	12	8

Which class had the best results on the problem? Explain your answer.

Syllogistic Reasoning Item:

Detective Puzzle

Suppose you are a detective and you have the following information about a burglary.

Either the burglar came in a car or the witness was mistaken.

If the burglar had an accomplice, then he came in a car.

The burglar did not have an accomplice and he didn't have a key to the apartment or the burglar had an accomplice and he had a key to the apartment.

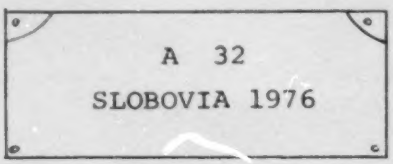
It has been proved that the burglar had a key to the apartment.

What conclusions can you make about coming in a car, the witness, and having an accomplice?

Combinatorial Thinking Item:

License Plate Puzzle

In a small foreign country, automobile license plates have a letter followed by two numbers.

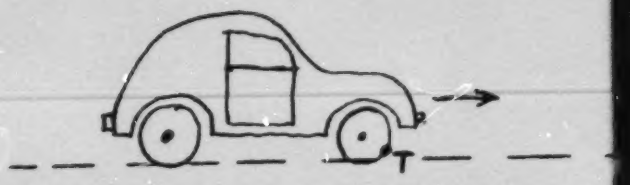


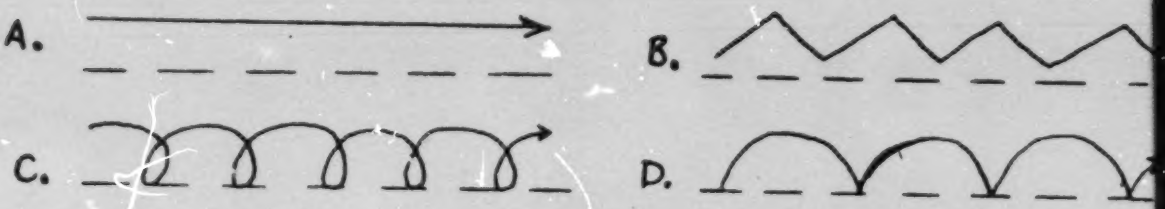
The only letters used are A and B. The numbers can be 1, 2, or 3. How many different license plates can be made? Describe how you figured it out.

Field Dependence Item

Tire Puzzle

As a car drives along the highway, its tires turn around in a clockwise direction. Imagine a tack (T) is stuck on the front tire. What would be the path of the tack as the car goes forward and the tire goes around? Choose the diagram that best show the tack's path.





Answer: _____

Please explain your choice.

Results

A 2 x 2 (treatment by cognitive level) design was used in the analysis of achievement and retention data. Subjects were classified as concrete if their score on the formal thinking test was less than four. As a result, 56 subjects were classified as concrete and 33 formal, which means that 63 percent of the subjects were concrete, supporting previous research results.

The means and standard deviations on the dependent measures for each treatment group are given in Table 2.

- - - - - - - - - - - - - - -
Insert Table 2 about here
- - - - - - - - - - - - - - -

A separate unequal cells, fixed effects ANOVA was computed for each of the dependent variables. A summary of the ANOVA F-ratios is given in Table 3.

- - - - - - - - - - - - - - -
Insert Table 3 about here
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None of the treatment main effect measures were significant. Only the posttest results were significant for the main effect of cognitive level, favoring the formal group. Several interactions were significant, however, and as can be seen from Figures 1a to 1d, the nature of the interactions were opposite those hypothesized. As the figures show,

- - - - - - - - - - - - - - - - - - -
Insert Figure 1a - 1d about here
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concrete subjects performed better with a lecture approach and formal subjects did better with a laboratory approach. The interactions were all disordinal and were more pronounced for the Retention test that the Posttest. On the Retention test, the concrete subjects in the lecture group actually increased their mean score over the posttest, while the formal laboratory group maintained about the same mean, possibly due to a ceiling effect. The results appear to indicate that both for short and long-term retention, concrete subjects did better in a lecture approach and formal subjects did better in a laboratory setting.

Discussion

A number of explanations are possible for these results. First, the laboratory lessons may have required a high level of reasoning ability in order to attain maximum benefit. Students worked largely on their own, requiring inferences about procedures and conclusions to be drawn. Second, the content of the units may not have been highly formal in nature, making a lecture approach the most efficient way to present most of the topics. The function of the laboratory method in this case might have been to provide motivation and involvement for formal thinkers, helping them to achieve at their maximal level. The formal subjects in the lecture group may have been disinterested, resulting in lower achievement.

Several potentially important factors emerge from the study. First, in selecting an instructional approach, the type of material must be considered. Concepts that are not formal may be best presented via lecture, unless high ability students need a more enriched learning environment. Also, in using a math laboratory approach for concrete subjects, care must be taken to make the procedures simple enough to produce benefit from experience with concrete materials. Perhaps a teacher-directed laboratory is more appropriate for concrete subjects, in which the structures to be formed are emphasized by the teacher, while the students experience the concrete manipulation.

Further research is necessary, especially in view of the fact that so many of these students do not reason formally in all areas. Many programs for prospective teachers use the laboratory approach. More study must be done to determine the extent of individual or group work that these students should do.

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Table 1

Content Outline of Instructional Units

Unit	Outline	Laboratory Manipulatives
I Reasoning	Inductive and deductive reasoning, informal proof	Unit cubes, cuisenaire rods
II Numeration*	Decimal and non-decimal numerals, operations	Multibase blocks
III Natural Numbers	Properties of natural number operations	Cuisenaire rods, pegboard, unit cubes
IV Whole Numbers	Algorithms, factors, primes, GCF, LCM, inequality	Balance beam, unit cubes, cuisenaire rods
V Number systems	Modular arithmetic, symmetry groups abstract operations	"clocks", cuisenaire rods, cardboard figures

*This unit was used to familiarize students with the laboratory approach. No achievement data was obtained.

Table 2

Cell Means and Standard Deviations[a]
for Achievement Tests

Treatment Group	N	Unit II[b]	Unit III	Unit IV	Unit V	Posttest[c]	Retention[c]
Lecture	41	8.95	8.49	7.76	7.90	39.44	41.74
		(1.43)	(1.38)	(2.09)	(1.94)	(6.42)	(6.25)
Concrete	28	9.14	8.53	7.85	7.67	39.46	42.69
		(1.20)	(1.40)	(2.35)	(2.12)	(5.65)	(3.94)
Formal	13	8.53	8.38	7.54	8.38	39.38	39.69
		(1.81)	(1.38)	(1.45)	(1.44)	(8.09)	(8.27)
Laboratory	48	9.14	9.04	7.08	7.26	39.94	40.91
		(1.03)	(1.35)	(2.53)	(2.78)	(6.95)	(9.34)
Concrete	28	8.85	8.64	6.39	7.17	37.57	39.05
		(1.78)	(1.54)	(2.89)	(2.80)	(7.82)	(11.28)
Formal	20	9.55	9.60	8.05	7.35	43.25	43.50
		(.60)	(.75)	(1.50)	(2.83)	(3.58)	(4.79)

[a]Standard deviations in parentheses

[b]Total possible score on all unit tests: 10

[c]Total possible score: 50

Table 3

Analysis of Variance Summary

for Achievement Tests

Source	df	Unit II	Unit III	Unit IV	Unit V	Posttest	Retention
Treatment	1	.50	3.15	2.37	1.74	.02	.32
Cog. Level	1	.25	2.14	2.54	.55	5.02**	.49
Interaction	1	5.86**	3.48	3.77*	.24	4.01*	4.44*
Residual	85						

* $p < .05$

** $p < .025$

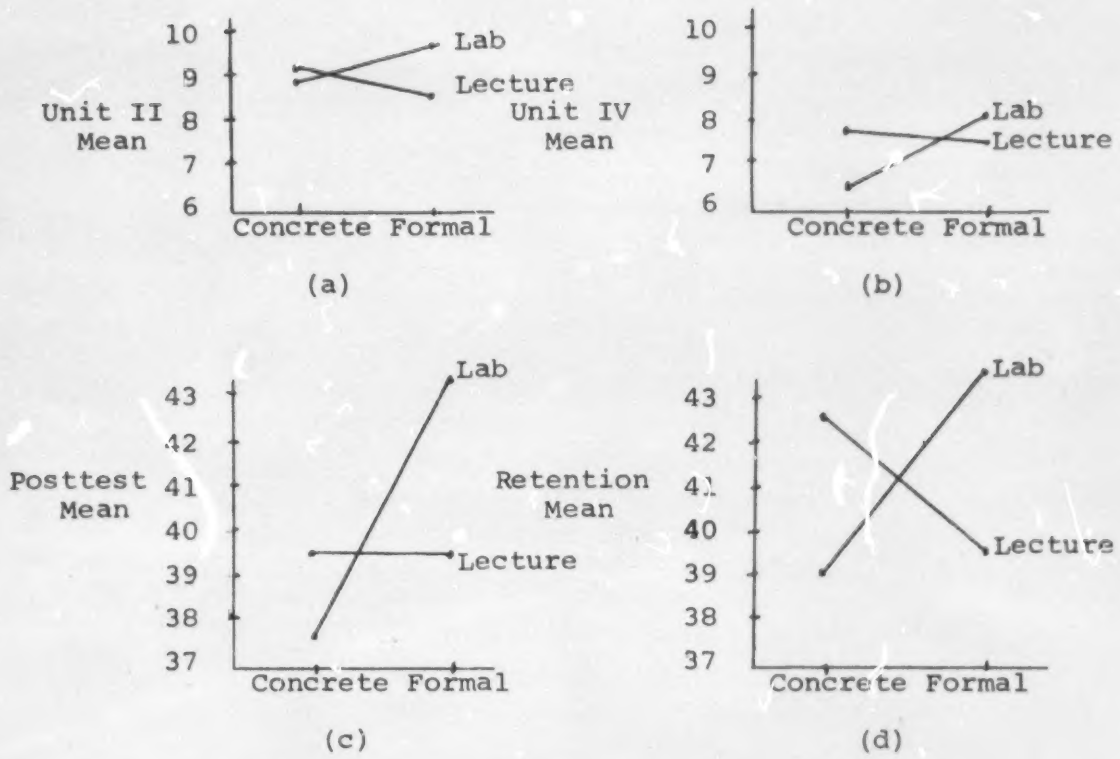

Figure 1. Diagrams of Treatment x Level Interactions.